Rare event simulation using Hamiltonian Monte-Carlo and Sequential Monte Carlo (SMC)

Risk Forum 2018

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Plan

Rare event simulation using Hamiltonian Monte-Carlo and Sequential Monte Carlo (SMC)

- Introduction and state-of-the art of particle methods
- Hamiltonian flow Monte Carlo methods
- Application to barrier option pricing

Rare Event Probability Estimation

Given some stochastic process $\{X_t\}_{t\geq 0}$, for any test function f, compute a rare event probability: $P(f(X_T) \geq b) = \mathbb{E}^{\mathbb{P}}[\mathbb{1}_{f(X_T)\geq b}]$.

- Crude Monte Carlo (CMC):

  Simulate M realizations of a random variable $X^{(m)}$ and get the estimate:

$$P(f(X_T) \geq b) \approx \frac{1}{M}\sum_{m=1}^{M} \mathbb{1}_{f(X_T^{(m)})\geq b} \tag{1}$$

  ⚠ A high number of samples is nedeed, especially with fat tailed distributions.

- Refined approaches
  - Importance sampling (IS), large deviations theory, EVT, line sampling
  - Sequential Monte Carlo Methods (SMC)
    P. Del Moral et al.: Genealogical particle analysis of rare events
  - Sequential Markov Chain Monte Carlo Methods (SMCMC)

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Feynman-Kac Measures

A sequence of Feynman - Kac measures is defined for any test function f as:

$$\eta_n f = \frac{\int \dots \int f(x_n)\eta_0(dx_0)\prod_{i=1}^{n-1}\mathcal{M}_i(x_i, dx_{i+1})}{\int \dots \int \eta_0(dx_0)\prod_{i=1}^{n-1}\mathcal{M}_i(x_i, dx_{i+1})} \quad (n \in \mathbb{N})$$

- where $\{\mathcal{M}_n\}_{n\in\mathbb{N}}$ the set of unnormalized Markov transitions on some measurable state space $(X, \mathcal{X})$;
- η_0 is initial probability measure;
- $\eta_n^M f \xrightarrow{M\to\infty} \eta_n f$ (a.s) and $\sqrt{N}(\eta_n^M f - \eta_n f) \xrightarrow[M\to\infty]{\mathcal{L}} \mathcal{N}(0, \Sigma(f))$ where $\eta_n^M f = \sum_{m=1}^{M} \frac{\omega_n^{(m)} f(\xi_n^{(m)})}{\sum_{j=1}^{M} \omega_n^{(j)}}$.

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Sequential Monte Carlo

SMC allows to approximate the empirical measure by a set of weighted particles ξ, ω.

$$\{\xi_n^{(m)}, \omega_n^{(m)}\}_{m=1}^M$$

Selection ↓

$$\{\xi_n^{\mathcal{I}_n^M(m)}, \omega_n^{\mathcal{I}_n^M(m)}\}_{m=1}^M \qquad \mathcal{I}_n^M(m) \in [1, M]$$

Mutation ↓

$$\{\xi_{n+1}^{(m)}, \omega_{n+1}^{(m)}\}_{m=1}^M$$

$$\xi_{n+1}^{(m)} \sim K_n(\xi_n^{\mathcal{I}_n^M(m)}, \cdot)$$
$$\omega_{n+1}^{(m)} = \omega_n^{\mathcal{I}_n^M} W_n^{\mathcal{I}_n^M}(\xi_n^{\mathcal{I}_n^M(m)}, \xi_{n+1}^{(m)}), \text{ where}$$

- $K_n(\xi_n, \cdot)$ is the transition kernel
- W_n is the importance weight function such that

$$W_n(x, y) = \frac{Q_n(x, dy)}{K_n(x, dy)} \quad (x, y) \in \mathcal{X}^2 \tag{2}$$

Sequential Importance Resampling/Bootstrap Resampling

Initialization: M - #(simulations), N - #(time steps), η_0 - initial measure

for $m = 1, \dots, M$ **do**

  $\xi_0^{(m)} \sim \eta_0$;

  $\xi_1^{(m)} \sim \mathsf{K}_0(\xi_0^{(m)}, \cdot)$;

  $\omega_1^{(m)} = \omega_0^{(m)}(\xi_0^{(m)}, \xi_1^{(m)})$

end

for $n = 1, \dots, N$ **do**

  for $m = 1, \dots, M$ **do**

    if $n < N$ **then**

      Selection step:

      Sample indices $\mathscr{I}_{n-1}^M(m)$ according to sampling strategy $\mathscr{S}(\{\omega_{n-1}^{(m)}\}_{m=1}^M)$;

    end

    Mutation step:

    Generate $\xi_n^{(m)}$ from $\mathsf{K}_n(\cdot, \xi_{n-1}^{\mathscr{I}_{n-1}^M(m)})$ and set $\widehat{\xi}_n^{(m)} = (\widehat{\xi}_n^{(m)}, \xi_{n-1}^{(m)})$;

    Compute the weight: $\omega_n^{(m)}(\widehat{\xi}_n^{(m)}) = W_{n-1}(\xi_{n-1}^{\mathscr{I}_{n-1}^M(m)}, \xi_n^{\mathscr{I}_{n-1}^M(m)})$;

  end

end

Markov Chain Monte Carlo – Metropolis-Hastings Algorithm

MCMC: Given a target π, design a transition kernel $\mathscr{K}$ such that asymptotically:

$$\frac{1}{N}\sum_{j=1}^{N} f(X^{(j)}) \xrightarrow{n\to\infty} \int_{\mathscr{X}} f(x)\pi(x)dx \text{ and } X^{(j)} \sim \pi \tag{3}$$

Introduce a family of proposal distribution $\{q(x,\cdot), x \in \mathscr{X}\}$, i.e. $\int q(x,y)dy = 1$ for any $x \in \mathscr{X}$.

The main idea of MH MCMC is,

- given that the Markov Chain is $x \in \mathscr{X}$, to propose a new candidate from $q(x,\cdot)$,
- to accept new proposal with an acceptance probability $a(x,y)$, which insures that the invariant distribution of the transition kernel $\mathscr{K}$ is the target distribution $\pi(x)$

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"Sequential" Markov Chain Monte Carlo/MH

Initialization: N - #(time steps);
$X_{0,0}^{j} \sim \mathcal{K}_0(X_{0,0}^{j-1}.\cdot)$ with $\mathcal{K}_0$ is a π_0 invariant MCMC kernel;
for $n = 1, ..., N$ **do**
  for $j = 1, ..., M + M_b$ **do**
    Sample $X_{n,0:n}^{j} = \mathcal{K}_n(X_{n,0:n}^{j-1}, \cdot)$ with $\mathcal{K}_n$ is a π_n invariant MCMC kernel;
  end
  $\pi(x_{0:n}) \approx \frac{1}{M}\sum_{j=M_b+1}^{M_b+M} \delta_{X_{n,0:n}^{j}}(dx_{0:n})$
end

Algorithm1:MH algorithm for the kernel $\mathcal{K}$

Input: $X_{n,0:n}^{j-1}$
Compute the acceptance probability: $a(X_{n,0:n}, x^*) = 1 \wedge \frac{\pi(x^*)q(x^*, X_{n,0:n}^{j-1})}{\pi(X_{n,0:n}^{j-1})q(X_{n,0:n}, x^*)}$
assuming that $a(x, y) = 0$ if $\pi(x)q(x, y) = 0$;
Draw $u \sim \mathcal{U}nif(0, 1)$ and set $X_{n,0:n}^{j} = x^*$, if $u < a$, and $X_{n,0:n}^{j} = X_{n,0:n}^{j-1}$ otherwise;

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Sequential MCMC

Problem: Slow convergence to the invariant density, when naive MCMC MH kernels are used.

Solution: Use Metropolized Hamiltonian system's proposals as MCMC kernel by augmenting r.v. with an auxillary variables

- Recipe comes from statistical physics and mathematical mechanics
- Hamiltonian flow's symplecticness and energy-conservation property entails ergodicity
- We replace the exact flow by its numerical discretization, however, invariant measure is no longer preserved under the discrete flow.
- We "Metropolize" to enforce the desired target distribution thereby accounting for the energy error of the numerical flow and to have an invariant measure.
- Geometrically ergodic for regular potentials

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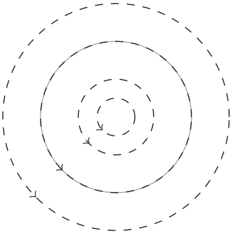
HMC Intro



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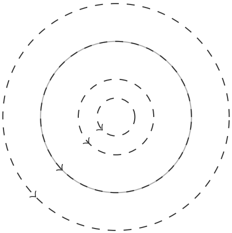

$X \rightarrow (X, P)$

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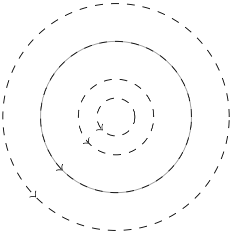

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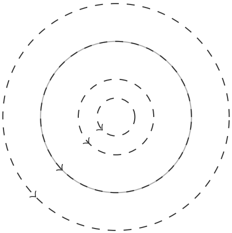

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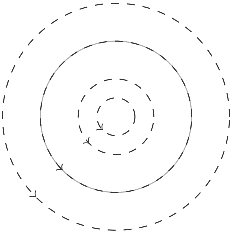

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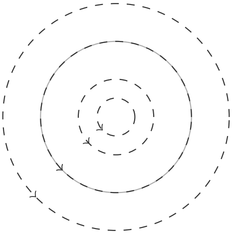

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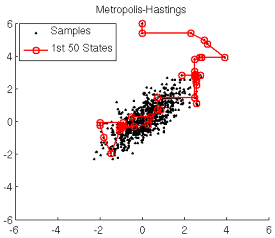

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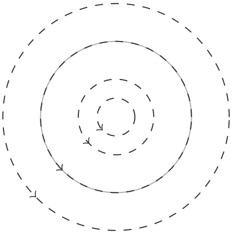

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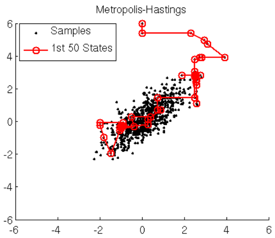

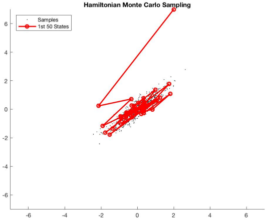

HMC. Example

- MCMC is useful when sampling a r.v. X having target density known up to a normalizing const:

$$\pi(X) = \frac{e^{-\Psi(X)}}{\mathcal{Z}} \quad \text{where} \tag{4}$$

- Consider a Gaussian log-likelihood function:

$$\log p(x_n|\theta) \propto \left(-\frac{1}{2}\left(\frac{x_n - k(x_{n-1},\theta)}{\sigma}\right)^2\right) = -\Psi(X) \tag{5}$$

- Extend the state space by a momentom r.v. P, so that the joint-distribution is given by:

$$\pi(X,P) = e^{-\mathcal{H}(X,P)} \propto e^{-\psi(X)} e^{-\frac{1}{2}P^T \mathfrak{M} P} \tag{6}$$

- Transition step: Solve the following ODE with initial conditions:

$$\begin{matrix} \partial_u P_u = -\nabla_x \Psi(X) & P_0 \sim e^{\frac{1}{2}P^T \mathfrak{M} P} & P* = P_L \\ \partial_u X_u = P & X_0 = X_0 & x^* = X_L \end{matrix} \tag{7}$$

- Acceptance step: accept x^*, P^* with probability:

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$$\log p(x_n|\theta) \propto \left(-\frac{1}{2}\left(\frac{x_n - k(x_{n-1},\theta)}{\sigma}\right)^2\right) = -\Psi(X) \tag{5}$$

- Extend the state space by a momentom r.v. P, so that the joint-distribution is given by:

$$\pi(X,P) = e^{-\mathscr{H}(X,P)} \propto e^{-\psi(X)} e^{-\frac{1}{2}P^T \mathfrak{M} P} \tag{6}$$

- Transition step: Solve the following ODE with initial conditions:

$$\begin{aligned} \partial_u P_u &= -\nabla_x \Psi(X) & P_0 &\sim e^{\frac{1}{2}P^T \mathfrak{M} P} & P* &= P_L \\ \partial_u X_u &= P & X_0 &= X_0 & x^* &= X_L \end{aligned} \tag{7}$$

- Acceptance step: accept x^*, P^* with probability:

$$a = min\left(1, \frac{\pi(x^*, P^*)}{\pi(X,P)} = \right) = 1 \wedge e^{-(\mathscr{H}(x^*,P^*) - \mathscr{H}(X,P))} \tag{8}$$

HMC. Example

- MCMC is useful when sampling a r.v. X having target density known up to a normalizing const:

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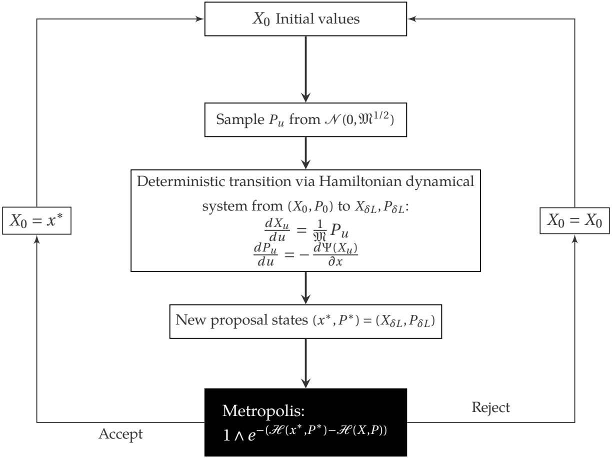
Hamiltonian Flow Monte Carlo Scheme



HMC in Rare Event Setting

- We compute the loglikelihood and its derivative, sample momentum variables
- Generate new proposal x^* by simulating Hamiltonian dynamics;
- Accept new proposals $X_{n+1} = x^*, P_{n+1} = P^*$ if acceptance probability satisfies $a > \mathscr{U}nif(0,1)$ and $x^* \in A$:
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Hamiltonian Monte Carlo Algorithm

Initialization: n_S - #(simulations), n_t - #(time steps)
for $n = 1, \ldots, n_t$ **do**
  for $m = 1, \ldots n_S$ **do**
    Generate $X_n^{(m)}$ from prior $\tilde{p}(X_0^{(m)}, \cdot)$;
    Simulate initial momentum $P_1^{(m)} \sim \mathcal{N}(0, I_M)$, set $x_H^{(m)} = X_n^{(m)}$ and run Hamiltonian flow:
    for $l_f = 1, \ldots L-1$ **do**

$$P_H^{(m)}((l_f + \tfrac{1}{2})\delta) = P_H^{(m)}(l_f) - \frac{\delta}{2}\frac{\partial \Psi}{\partial x_H}(x_H^{(m)}(l_f))$$
$$x_H^{(m)}((l_f+1)\delta) = x_H^{(m)}(l_f) + \delta P_H^{(m)}((l_f + \tfrac{1}{2})\delta) I_M^{-1}$$
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    end
    Set $x^* = x_H^{(m)}(L)$, $P^* = P_H^{(m)}(L)$ and calculate acceptance probability:
    $a = 1 \wedge e^{(-\mathcal{H}(x^*, P^*) + \mathcal{H}(x_H^{(m)}, P_H^{(m)}))\Delta t}$;
    Draw $\mathfrak{u} \sim \mathcal{U}\text{nif}(0,1)$ and set $X_n^j = x^*$, if $\mathfrak{u} < a$ and $x^* \in A$, and $X_n^j = X_n^{j-1}$ otherwise;
  end
end
Compute: $\hat{C}^{HFMC} = \frac{1}{n_S}\sum_{m=1}^{n_S}\left(f(X_{n_t}^{(m)})\prod_{n=1}^{n_t} e^{(-\mathcal{H}(X_{n+1}^{(m)}, P_{n+1}^{(m)}) + \mathcal{H}(X_n^{(m)}, P_n^{(m)})\Delta t}\mathbb{1}_{X_{n+1}^{(m)} \in A_n}\right)$

Application of HMC on a barrier option pricing

- $$C = e^{-(r-q)T}\mathbb{E}[g(X_{n_S})\prod_{n=1}^{n_t}\mathbb{1}_{X_{t\in[t_{n-1},t_n]}\in A_n}] \tag{9}$$

- $$A_n = \inf_{t_{n-1}\leq t\leq t_n}\{t : X_t > B\}$$

- HFMC estimator to compute DOC call option is given by:

$$\widehat{C}^{HFMC} = e^{-(r-q)T}\frac{1}{n_S}\sum_{m=1}^{n_S}\left(g(X^{(m)}_{n_t})\prod_{n=1}^{n_t}e^{(-\mathcal{H}(X^{(m)}_{n+1},P^{(m)}_{n+1})+\mathcal{H}(X^{(m)}_{n},P^{(m)}_{n})\Delta t}\mathbb{1}_{X^{(m)}_{n+1}\in A_n}\right) \tag{10}$$

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Application of HMC on a barrier option pricing, $B = 65$

Table: DOC Barrier option estimates statistics. $X_0 = 100$, $K = 100$, $r = 0.1, \sigma = 0.3$, $T = 1/2$, and $div = 0$; $\delta = 0.0001$, $B = 65$, #(Leap frog step): 40. True price: 10.9064, $N = 75000$, $M = 750$

Stat	MC	IPS	HFMC
St. dev.	0.062385996	0.044259477	0.038039517
RMSE	0.037561882	0.051285344	0.037561882
RRMSE	0.000355199	0.000240548	0.000129293
CPU time	2.2626	6.0322	7.6832
FOM	13475.2	10117.7	10711.0

Further research and improvements

- Combine with splitting algorithm

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Merci a tous pour votre attention !

Thanks for your attention!