Disastrous Defaults

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INTRODUCTION

Introduction

- This paper presents a framework aimed at analysing the asset pricing and macro implications of the existence of "systemic defaults".
- It is flexible and tractable enough to simultaneously replicate the price fluctuations of various far-out-of-the-money (disaster-exposed) credit and equity derivatives.
- Bringing (macro) structure to the model, we can extract information from asset prices, such as the expected influence of a systemic default on consumption or probabilities of financial meltdowns.

Introduction

- Disaster Risk (DR), defined as a sudden and dramatic decrease in output and consumption, helps solve many asset-pricing puzzles.

  [Rietz, 1988, Barro, 2006, Gabaix, 2012].
- Several contributions show that far-out-of-money credit and equity options provide useful information regarding DR.
- DR generally modelled as an exogenous event causing simultaneously
  - sharp decreases in economic output or consumption,
  - dramatic increases in the default probabilities of bond issuers and/or
  - dramatic decreases in the asset values of firms.
- But the default of a systemic entity is, by itself, (at least perceived as) a disaster:
  - Largest $\searrow$ in the U. of Michigan Consumer Sent. index: 09/2008. chart
  - This is at the core of novel regulations on SIFIs [Battiston et al., 2016, Brownlees and Engle, 2017].

This paper

- Structural no-arbitrage asset-pricing framework where the defaults of some entities, called systemic entities, have economy-wide effects.
- The default of a systemic entity

can have a negative effect on economic activity / consumption

$+$

is contagious (can provoke additional systemic defaults)

$\Rightarrow$ A systemic default is disastrous.

- The model is tractable. Closed-form formulas for various credit/equity options.
- The model captures the main fluctuations of prices of various disaster-exposed instruments (European data, 2006-2017):

  Credit Index swaps, Synthetic CDOs, far-out-of-the-money equity put options.

Results overview

- Assets exposed to disaster risk (systemic defaults) carry important credit risk premiums.

  [Credit risk premiums: prices/spreads diff. between observed prices/spreads and the prices/spreads that would prevail if agents were risk-neutral.]

- By nature, systemic defaults happen in bad states.

  $\Rightarrow$ A large part of the spreads of CDS written on systemic entities corresponds to risk premiums ($\approx$ 75% for the 10-year maturity).

- Joint modelling of macroeconomic variables and financial prices reveals expected macro impact of systemic events:

  $\Rightarrow$ A systemic default is anticipated to be followed by a 4% $\searrow$ in consumption.

- Systemic risk indicators = Probabilities of having more than 10% of defaults among the 125 iTraxx constituents within two years:
  - 6% in Sept. 2008 (Lehman bankruptcy)
  - 4% in late 2011 (euro-area sovereign debt crisis).

Synthetic view of connected literature

		This paper	[Seo and Wachter, 2018]	[Siriwardane, 2016]	[Barro and Liao, 2016]	[Collin-Dufresne et al., 2012]	[Christoffersen et al., 2017] (a)	[Christoffersen et al., 2017] (b)	[Coval et al., 2007]	[Longstaff and Rajan, 2008]	[Azizpour et al., 2011]	[Giesecke and Kim, 2011]
Disaster	Endogenous	✓										
	Exogenous	(✓)	✓	✓	✓	✓	✓	✓		✓	✓	✓
Structural	(Macro)	✓	✓	✓	✓		✓					
Asset class	Stock options	✓	✓	✓	✓	✓	✓					
	CDS/Bond spd	✓	✓			✓	✓	✓	✓	✓	✓	✓
	Tranches	✓	✓			✓		✓	✓	✓	✓	✓
Param.	Estimated	✓			✓	✓		✓	✓	✓	✓	✓
	Calibrated	(✓)	✓	✓			✓					
Period	Start	06	05	97	94	04		05	04	03	04	70
	End	17	08	14	15	08		07	06	05	07	08

MODEL AND PRICING

Model (1/4)

- $N_{j,t}$: number of Segment-j entities in default at date t.

  $$\left\{\begin{array}{lcl} \text{S1 and S2} & : & \text{Systemic entities (S1: Components of traded credit indices).} \\ \text{S3} & : & \text{Non-systemic entities.} \end{array}\right.$$

  S1: entities in traded index portfolios; S2: other systemic entities.

- N_t: Vector $N_t = [N_{1,t}, \dots, N_{J,t}]'$.

- $n_{j,t}$: Number of defaults in Segment j on date t, i.e. $n_{j,t} = N_{j,t} - N_{j,t-1}$.

- x_t and y_t: $x_t \geq 0$, $y_t \geq 0$,
  Exogenous processes with Gamma-type transition distributions. Dynamics:

  $$\left\{\begin{array}{lcl} x_t - \mu_x & = & \rho_x(x_{t-1} - \mu_x) + \sigma_{x,t}\varepsilon_{x,t} \\ y_t - x_t & = & \rho_y(y_{t-1} - x_{t-1}) + \sigma_{y,t}\varepsilon_{y,t}, \end{array}\right. \quad (1)$$

  (V-ARG: [Gouriéroux and Jasiak, 2006] or [Monfort et al., 2017])

Model (2/4)

- For any process k_t (say), we use the notation $\underline{k_t} = \{k_t, k_{t-1}, \dots\}$.
- Conditional distribution of the number of defaults:

$$n_{j,t+1}|\underline{x_{t+1}}, \underline{y_{t+1}}, \underline{N_t} \quad \sim \quad \mathcal{P}oisson(\beta_j y_{t+1} + c_j n_t^s), \tag{2}$$

  where $n_t^s = n_{1,t} + n_{2,t}$, i.e. n_t^s = nb of systemic defaults on date t.
- If $c_j > 0$:
  Systemic defaults on date t increases the conditional probability of having defaults in Segment j on the next date.

$\Rightarrow$ Systemic defaults are infectious [Davis and Lo, 2001], or contagious.

Model (3/4)

- $\Delta c_t = \log(C_t/C_{t-1})$: Log growth rate of per capita consumption. Δc_t follows:

$$\Delta c_t = \mu_{c,0} + \mu_{c,x} x_t + \mu_{c,y} y_t + \mu_{c,w} w_t. \quad (3)$$

where w_t depends on systemic defaults:

$$w_t | \underline{x_t}, \underline{y_t}, \underline{N_t} \sim \gamma_0(\xi_w n^s_{t-1}, \mu_w). \quad (4)$$

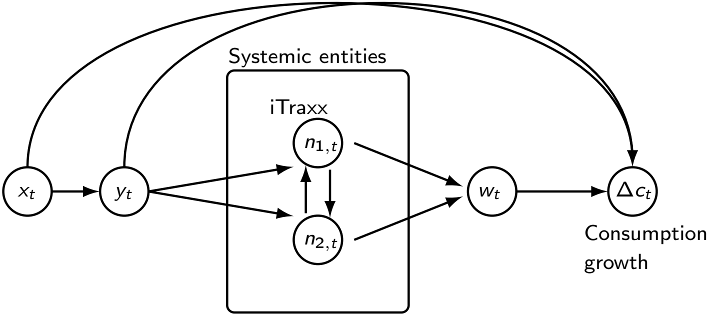
- γ_0 is a distribution featuring a point mass at zero [Monfort et al., 2017], i.e.

$$w_t | \underline{x_t}, \underline{y_t}, \underline{N_t} \quad \begin{cases} \sim \gamma(\mathcal{Z}_t, \mu_w), \text{ with } \mathcal{Z}_t \sim i.i.d.\, \mathcal{P}(\xi_w n^s_{t-1}) & \text{if} \quad n^s_{t-1} > 0 \\ = 0 & \text{if} \quad n^s_{t-1} = 0. \end{cases}$$

$\Rightarrow$ The conditional probability that $w_t = 0$ is $\exp(-\xi_w n^s_{t-1})$,
$w_t = 0$ as long as there has been no systemic defaults in the previous period.

- If $\mu_{c,w} < 0$ and $|\mu_{c,w}|$ is large, then
systemic defaults can give rise to "disastrous" decreases in C_t.

Model (4/4)


Systemic entities
iTraxx
$n_{1,t}$
$n_{2,t}$
x_t
y_t
w_t
Δc_t
Consumption growth

Pricing formulas (1/2)

- Agents feature Epstein-Zin preferences, with a unit elasticity of intertemporal substitution (EIS). [Piazzesi and Schneider, 2007, Seo and Wachter, 2018].
- The time-t utility of a consumption stream ($C_t = \exp(c_t)$) is recursively defined by

$$u_t \quad = \quad (1-\delta)c_t + \frac{\delta}{1-\gamma}\log\left(\mathbb{E}_t \exp\left[(1-\gamma)u_{t+1}\right]\right). \tag{5}$$

  where δ denotes the time discount factor and γ is the risk aversion parameter.
- $X_t = [x_t, y_t, w_t, N_t', N_{t-1}']'$ is affine $\Rightarrow$ we can solve for u_t $\Rightarrow$ The s.d.f. is of the form:

$$M_{t,t+1} = \exp[-(\eta_0 + \eta_1' X_t) + \pi' X_{t+1} - \psi(\pi, X_t)],$$

  where $\psi(\pi, X_t)$ is the condit. log-Laplace transform of X_t, i.e. $\mathbb{E}_t(e^{u' X_{t+1}}) = e^{\psi(u, X_t)}$.
- The risk-neutral measure is then defined by means of the change of probability:

$$\left(\frac{d\mathbb{Q}}{d\mathbb{P}}\right)_{t,t+1} = \frac{M_{t,t+1}}{\mathbb{E}_t(M_{t,t+1})} = \exp[\pi' X_{t+1} - \psi(\pi, X_t)]. \tag{6}$$

Pricing formulas (2/2)

Credit instruments:

- The risk-neutral dynamics of the number of defaults is implied by eq. (6).

⇒ Formulas to price CDS, Credit Index swaps (CIS) and synthetic CDO. ▸ CDO

CDS: protection payoff > 0, when the entity on which the CDS is written defaults.
CIS: protection payoff > 0, when one entity of the underlying portfolio defaults.
CDO: protection payoff > 0, when one entity of the underlying portfolio defaults, given that losses are in a given interval $[a, b]$ (e.g. $[a, b] = [3\%, 6\%]$).

- Typical credit indices: iTraxx (Europe) and CDX (U.S.).

Equity products:

- Model assumption: The dividend growth rate of a stock index is affine in X_t:

$$g_{d,t} = \mu_{d,0} + \mu_{d,x} x_t + \mu_{d,y} y_t + \mu_{d,w} w_t.$$

- X_t affine ⇒ Closed-form solutions for the stock index price, puts and calls. [Bansal and Yaron, 2004, Eraker, 2008]
- Typical equity indices: EUROSTOXX (Europe) and S&P (U.S.).

ESTIMATION AND RESULTS

Data

- Data: January 2006 to September 2017 at a bi-monthly frequency.
- Credit derivatives:
  - iTraxx Europe main index. 125 large European firms, whose credit default swaps are actively traded.
  - Credit index swap (CIS). Maturities: 3, 5, 7 and 10 years.
  - CDOs: maturities of 3, 5 and 7 years and, for each maturity, 5 tranches: 0%-3%, 3%-6%, 6%-9%, 9%-12% and 12%-22%.
- Equity derivatives:
  - Equity put options written on the EUROSTOXX 50.
    Maturities of 6 and 12 months,
    Strike = 70% of equity index,
    i.e. options protecting against larger-than-30% falls in the equity index.

An estimation approach that benefits from model tractability

- Γ_t: vector of observed prices (4 CIS, 15 CDO, 2 equity put options).
- Over our estimation period $n_t^s = 0 \Rightarrow$ the model predicts that these prices are functions of $z_t = [x_t, y_t]'$ and of Θ (vector of model parameters).

- Measurement equations (#21):

$$\Gamma_t = F(z_t; \Theta) + \epsilon_t, \tag{7}$$

where ϵ_t are measurement errors, $\epsilon_t \sim i.i.d.\mathcal{N}(0, \Sigma_\epsilon)$.

- Transition equations (#2) = dynamics of z_t:

$$z_{t+1} = \mu_z + \Phi_z z_t + \Sigma_z(z_t)\xi_{t+1}, \tag{8}$$

where ξ_{t+1} is a martingale difference sequence with $\mathbb{V}ar_t(\xi_{t+1}) = Id$.

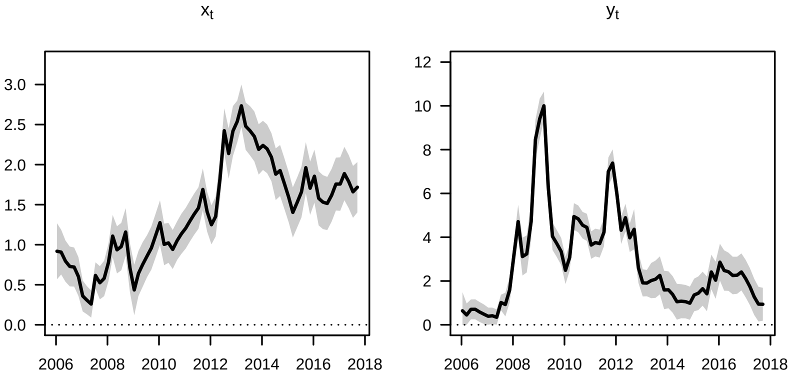
- Some parameters are calibrated (see next slide).

  Remaining parameters are estimated by maximizing the approximate log-likelihood computed by an Extended Kalman filter applied on the state-space model (7)-(8).

Panel (a) – Calibrated parameters			Panel (b) – Estimated parameters		
γ		3	$c_i \quad i \in \{1,2\}$		0.35
δ		0.997			
EIS		1.00	$\beta_i \quad i \in \{1,2\}$	$(\times 10^2)$	1.81
			μ_w		118.07
$\mathbb{E}(\Delta c_t)$	$(\times 6)$	1.50%	ξ_w		0.13
$\mathbb{E}(g_{d,t})$	$(\times 6)$	1.50%			
			μ_x	$(\times 10^2)$	1.22
			μ_y	$(\times 10^2)$	5.63
			ρ_x		0.978
			ρ_y		0.858
			$\mu_{c,x}$	$(\times 10^5)$	-0.08
			$\mu_{c,y}$	$(\times 10^5)$	-12.61
			$\mu_{c,w}$	$(\times 10^4)$	-8.17
			$\mu_{d,x}$	$(\times 10^5)$	-0.00
			$\mu_{d,y}$	$(\times 10^5)$	-0.00
			$\mu_{d,w}$	$(\times 10^4)$	-16.15

This table presents the model parameterisation. $\mathbb{E}(\Delta c_t)$ is multiplied by 6 so as to be expressed in annualised terms. The parameterisation is such that $\mathbb{E}(x_t) = \mathbb{E}(y_t) = 1$. Panel (b) reports parameters estimated by maximising an approximation of the log-likelihood associated with the state-space model defined by measurement equations (7) and transition equations (8).

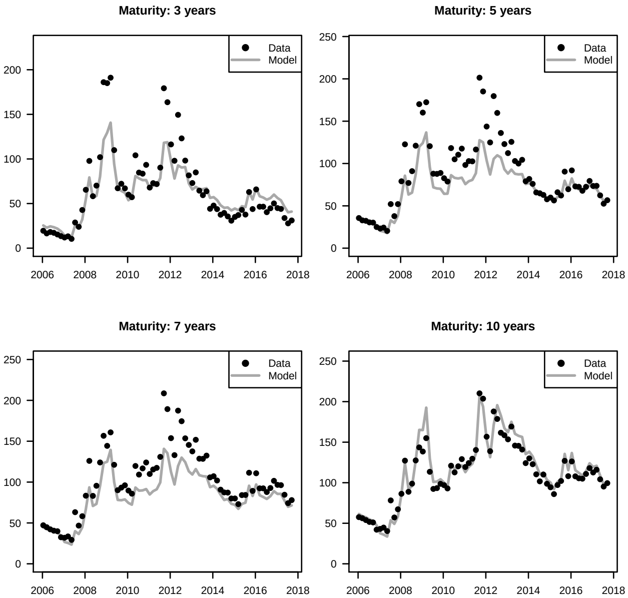
Estimated factors x_t and y_t



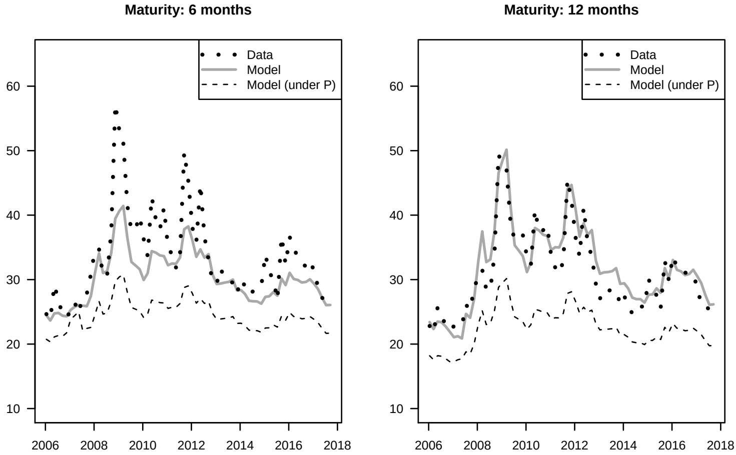
$$\begin{cases} x_t - \mu_x = \rho_x(x_{t-1} - \mu_x) + \sigma_{x,t}\varepsilon_{x,t} \\ y_t - x_t = \rho_y(y_{t-1} - x_{t-1}) + \sigma_{y,t}\varepsilon_{y,t}. \end{cases}$$

Because $\rho_y < \rho_x \approx 1$, x_t can be interpreted as the "trend" of y_t.

Fit of iTraxx index swap spreads



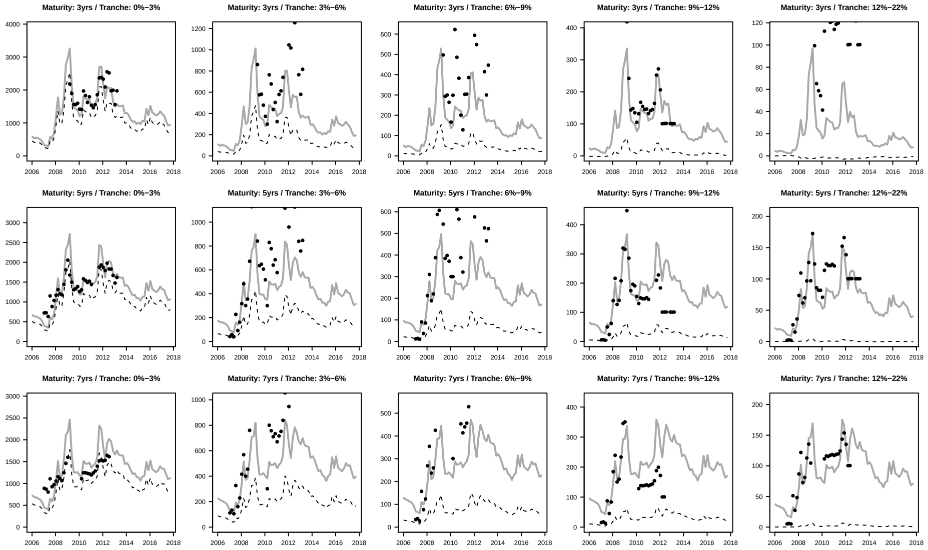
Fit of stock options (strike = 70% of spot index value)



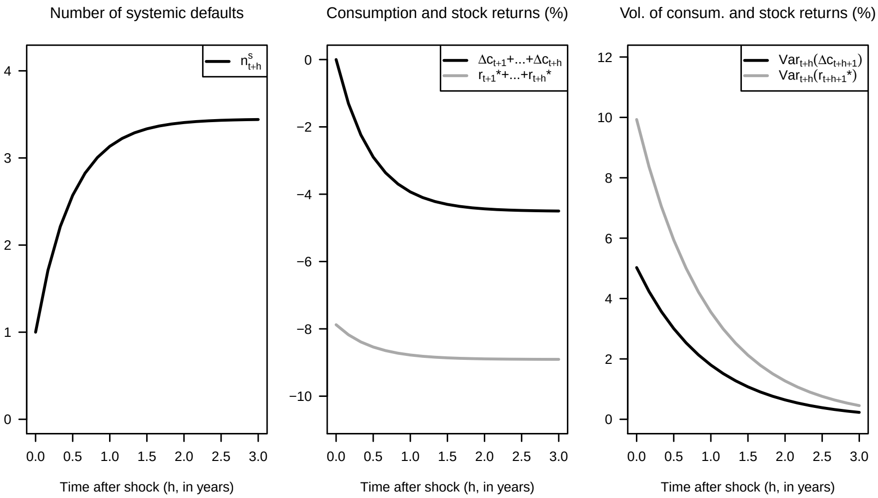
Dashed line: Implied vol. that would be observed if agents were risk-neutral.

(⇒ Spread between grey line and dashed line = measure of variance risk premium.)

Fit of iTraxx tranches (grey:fitted, dashed: without risk premiums))



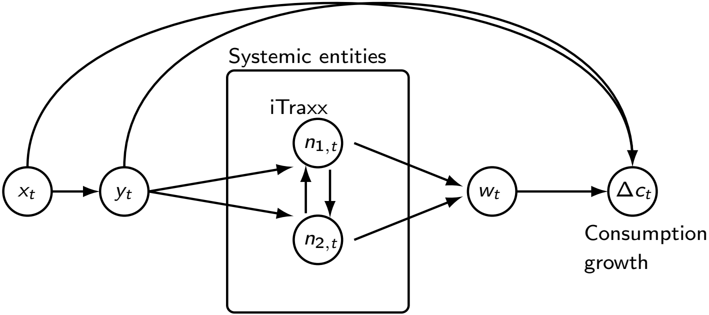
Responses to an unexpected default of a systemic entity



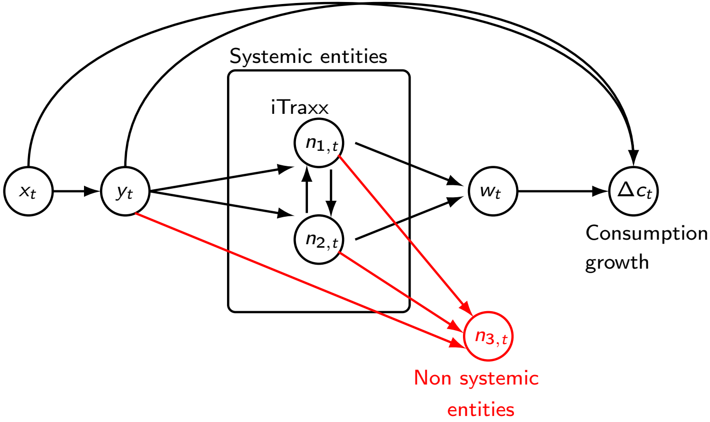
Responses are in percent.

EXTENDED MODEL

Adding non-systemic entities (Segment 3)



Adding non-systemic entities (Segment 3)



Adding non-systemic entities (Segment 3)

- To estimate the model, we just need to consider systemic segments (S1 and S2).
- Once estimated, the model can be used to study non-systemic-related credit instruments (S3).
- We consider different exposures to standard risk (β_3) and to systemic risk (c_3):

$$n_{3,t+1}|\underline{x_{t+1}}, \underline{y_{t+1}}, \underline{N_t} \sim \mathcal{P}(\beta_3 y_{t+1} + c_3 n_t^s)$$

- Slide 23:

  Credit spreads for non-systemic entities that would have the same average proba. of default (PD) than our systemic entities, but are exposed only to y_t (i.e. $c_3 = 0$)

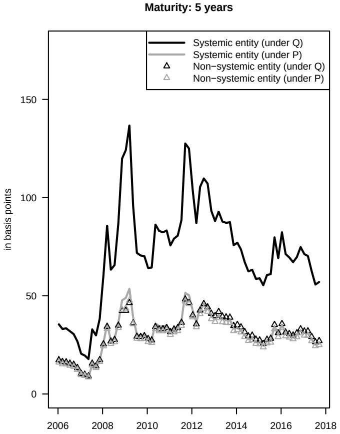
  $\Rightarrow$ Almost no credit risk premiums ($\mathbb{Q}$ spreads $\approx$ $\mathbb{P}$ spreads).

- Slide 24:

  Ratios between $\mathbb{Q}$ spreads and $\mathbb{P}$ spreads depending on (β_3, c_3) exposures.

  $\Rightarrow$ For a given PD, the larger the exposure to systemic risk, the higher the risk premiums (and the higher the CDS spread).

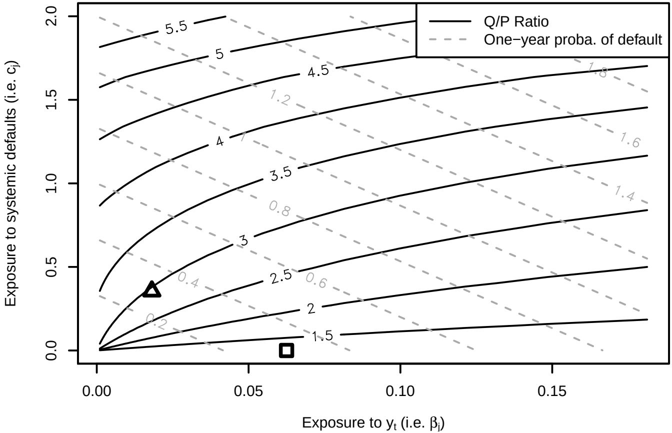
Differences in Risk Premiums between systemic and non-systemic entities



This figure shows CDS spreads written on systemic entities (solid lines) and non-systemic entities (triangles).

In grey: CDS spreads without risk premiums $\Rightarrow$ spds between black and grey curves = risk premiums.

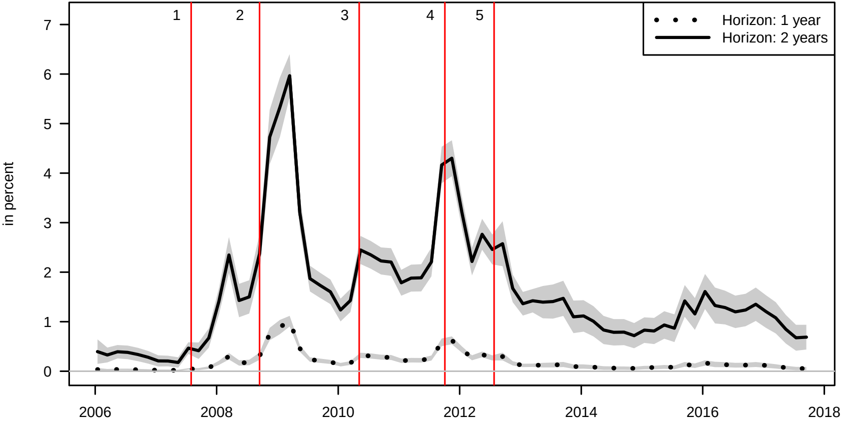
Impact of exposures to the exogenous factor y_t and to the number of systemic defaults n_t^s on the average size of credit risk premiums



We have: $n_{j,t+1}|\underline{x_{t+1}}, \underline{y_{t+1}}, \underline{N_t} \sim \mathcal{P}(\beta_j y_{t+1} + c_j n_t^s)$.

For entities represented by the triangle, the CDS spread is three times higher than the expected loss (solid black curve) and the probability of default is on average 0.35% (dashed grey curve)

Probability that at least 10% of iTraxx constituents default



CONCLUSION

Concluding remarks

- We introduce a structural no-arbitrage model allowing to study the pricing and macro implications of the existence of disastrous defaults.
- Being tractable, the model can be estimated on cross-sections of equity and credit options including CDS, Credit Index swaps and synthetic CDOs.
- We obtain estimates of risk premiums for all considered instruments. Risk premiums reflect the aversion of investors for systemic risk.

  Ex.: If agents were not risk-averse, 10-year CDS written on systemic entities would be 75% cheaper.
- The fraction of risk premiums in CDS or CDO spreads is relatively higher for instruments that are more exposed to systemic risk.
- The estimated model suggest that a systemic default is anticipated to be followed by a 4% $\searrow$ in consumption (i.e. a systemic default is disastrous).

Thank you!

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Synthetic Collateralised Debt Obligations (CDOs)

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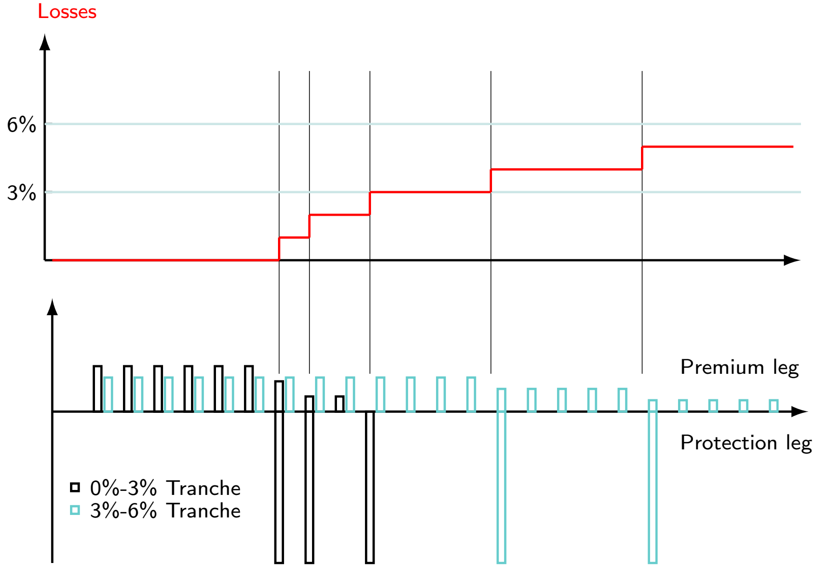

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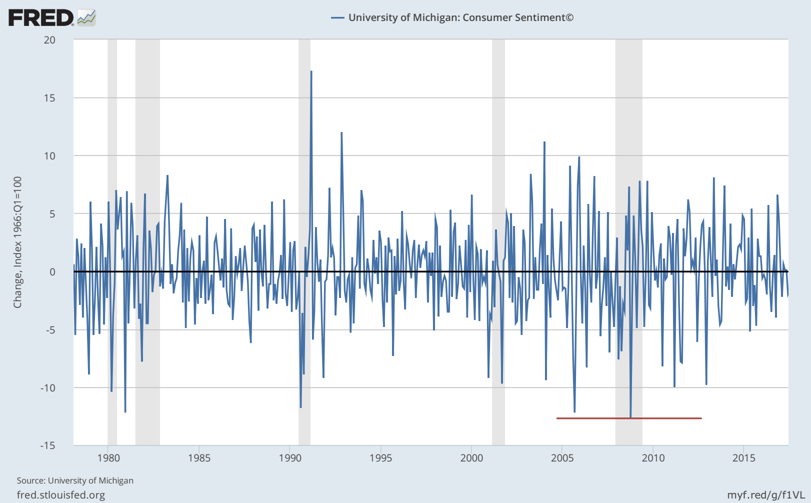

(Lowest value reached on September 2008)