11th Financial Risks International Forum

Valuing Life as an Asset, as a Statistic and at Gunpoint

Julien Hugonnier[1] Florian Pelgrin[2] Pascal St-Amour[3]

[1]École Polytechnique Fédérale de Lausanne, CEPR and Swiss Finance Institute

[2]EDHEC Business School, CIRANO and Institut Louis Bachelier

[3]University of Lausanne, Faculty of Business and Economics (HEC) and Swiss Finance Institute

Paris, March 26, 2018

1. Introduction
Motivation and outline

Different valuation methods to evaluate the price of human life

- **Cost of indemnisation** : Compensation paid by insurance companies to cover insured losses ;
- **Human capital life value** : Prejudice caused to society by the death/injury of an individual (occupational, end-users' wrongful death litigation)—PV of the net cash flow associated with human capital

$$v^j_{h,t} = \mathrm{E}_t \sum_{s=0}^{T_m} \left(\frac{1}{1+r} \right)^s D_{t+s}$$

where D_{t+s} is the (market value) of the net dividend $t+s$—marketed labor income *minus* all expenses to maintain human capital.

- **Value of a statistical life** :

  - ✓ Based on individual Willingness-To-Pay (WTP) to avoid small increases in exposure to death risk

  - ✓ Aggregation of individual WTP $\Rightarrow$ Collective WTP to save one unidentified (i.e. statistical) life.

  - ✓ **Example** : Suppose a population of size n and a change $\Delta = 1/n$ in death risk exposure. All agents are individually willing to pay $v_i(\Delta) = 1'000$. The **empirical** VSL is the **collective** WTP :

$$v_s = \sum_{i=1}^{1000} v_i(\Delta) = \frac{v_i}{\Delta} = 1\text{MM\$}.$$

    Note that the **theoretical VSL** is the negative of the MRS between the probability of death and wealth (or the marginal WTP) and is not observable !

TABLE : Comparison HK value and VSL (in $)

	Average HK value	Average VSL
Health Status		
Poor	249 532	2 719 261
Fair	318 865	5 126 530
Good	388 198	7 239 006
Very Good	457 531	9 518 831
Excellent	526 864	11 864 750
All individuals		
Mean	420 729	3 351 519
Median	457 731	8 803 507

VSL is 10-20 times larger than the HK value of life !

Main research question : How can we explain and assess this large discrepancy of valuation methods ?

♯ 1 Can we provide a reasonable metric for the value of life against which the two alternatives can be gauged ?

- ✓ Define third life value as **benchmark** : Gunpoint Value (GPV) overview

♯ 2 Can a common theoretical and empirical framework help in rationalizing the differences between the HK value and the VSL ?

- ✓ Provide **common** theoretical framework for HK, WTP, GPV and VSL.
- ✓ **Closed-form solutions** for HK, WTP, VSL and GPV values of life to evaluate :
  - Role of preferences, technological, distributional parameters.
  - Role of wealth, human capital
  - Shape of WTP.
  - Aggregation issues.
- ✓ **Structurally** estimate WTP, three values with **common** data set.

♯ 3 What lessons can we learn about the interpretation and applicability of the alternative measures in pricing the economic value of a human life ?

- ✓ HK and GPV directly compute the value of a whole life, rather than using a linear extrapolation to obtain a unitary life value (VSL) ;
- ✓ VSL should *not* be interpreted as the value of a given human being (Schelling, 1968)—rather a local measure of a rate of substitution between wealth and life ;
- ✓ VSL is appropriate when computing a collective value on small *indiscriminate* reductions on mortality for which society will ultimately end up paying the costs (e.g., public's safety) ;
- ✓ HK and GPV appear the better alternatives for wrongful death litigation or curative *vs* terminal care decisions.

Road map

2. A common framework for life valuation
Economic environment

- Assumption : Planning horizon is limited by a stochastic age at death T^m :

$$\lim_{h \to 0} \mathtt{Pr}\left[T_m \in (t, t+h] \mid T_m > t\right] = \lambda_m$$

  such that the probability of death by age t (death risk exposure) is monotone increasing in λ_m :

$$\begin{aligned} \mathcal{P}(t) &= \mathtt{Pr}\left(T_m \leq t\right) \\ &= 1 - \exp\left(-\lambda_m t\right) \end{aligned}$$

Note : Changes in death risk exposure $\mathcal{P} \Leftrightarrow$ changes in the instantaneous death intensity λ_m

- Law of motion H_t

$$\mathrm{d}H_t = \left[I_t^{\alpha} H_t^{1-\alpha} - \delta H_t\right] \mathrm{d}t - \phi H_t \mathrm{d}Q_{st}$$

where $\mathrm{d}Q_{st}$ is a Poisson depreciation (morbidity) shock with constant intensity λ_{s0} that further depreciates the health stock by $\phi \in (0,1)$.

- Budget constraint and income : Individuals can trade in two risky assets to smooth out shocks to consumption—stock and insurance *against* health depreciation

$$\begin{aligned} dW_t &= [rW_t + Y_t - c_t - I_t]\,\mathrm{d}t + \pi_t \sigma_S\,[\mathrm{d}Z_t + \theta \mathrm{d}t] \\ &\quad + x_t\,[\mathrm{d}Q_{st} - \lambda_{s0}\mathrm{d}t]\,, \\ Y_t &= y + \beta H_t, \end{aligned}$$

where π_t denotes the risky portfolio and x_t the units of an actuarially-fair insurance.

Preferences

Stochastic Differential Utility (Duffie and Epstein, 1992) :

- Disentangle risk aversion γ from intertemporal elasticity of substitution ε ;
- Minimum subsistence consumption a ;
- Preference for life over death ;
- $V^m \equiv 0$;

$$U_t = E_t \int_t^{T_m} \left(f(c_\tau, U_\tau) - \frac{\gamma |\sigma_\tau(U)|^2}{2U_\tau} \right) \mathrm{d}\tau,$$

where the age of death T_m is the first occurrence of a Poisson process with constant intensity λ_m and the Kreps-Porteus aggregator is :

$$f(c_t, U_t) = \frac{\rho\, U_t}{1 - 1/\varepsilon} \left(\left(\frac{c_t - a}{U_t} \right)^{1-\frac{1}{\varepsilon}} - 1 \right).$$

Optimal allocation V, c, I, π, x

Theorem

Optima closed-form allocations are given by :

$$c_t = a + A(\lambda_m)N(W_t, H_t)$$

$$\pi_t = \frac{\theta}{\gamma\sigma_S} N(W_t, H_t)$$

$$x_t = \phi P(H_t)$$

$$I_t = \left(\alpha^{\frac{1}{1-\alpha}} B^{\frac{\alpha}{1-\alpha}}\right) P(H_t)$$

$$V_t(W_t, H_t, \lambda_m) = \Theta(\lambda_m)N(W_t, H_t)$$

- ✓ $N(W, H) = \underbrace{W}_{\text{Fin. Wealth}} + \underbrace{(y-a)/r}_{\text{NPV of fixed inc. stream}} + \underbrace{P(H)}_{\text{Shadow value = BH}}$
- ✓ Marginal value of N : $\Theta(\lambda_m) = \tilde{\rho}A(\lambda_{m0})^{\frac{1}{1-\varepsilon}} \geq 0$
- ✓ MPC : $A(\lambda_m) = \varepsilon\rho + (1-\varepsilon)\left(r - \lambda_m + 0.5\theta^2/\gamma\right) \geq 0$

3. Values of life
Human capital value of life

Proposition

The HK value of life $v_{h,t} = v_h(W_t, H_t, \mathcal{P}_0)$ is the expected discounted present value over stochastic horizon T_m of labor revenue flows, net of investment costs,

$$
\begin{aligned}
v_{h,t} &= E_t \int_0^{T^m} m_{t,\tau} \left[Y(H_\tau^*) - I_\tau^* \right] \mathrm{d}\tau \\
&= E_t \int_0^{T^m} m_{t,\tau} \left[y + (\beta H_\tau^* - I_\tau^*) \right] \mathrm{d}\tau
\end{aligned}
$$

where $m_{t,\tau} = m_\tau / m_t$ *with* $m_t = \exp\left(-rt - \theta Z_t - 0.5\theta^2 t\right)$, *and writes*

$$
v_h(H, \lambda_m) = C_0 \frac{y}{r} + C_1 P(H)
$$

$$
\textit{with} \quad C_0 = \frac{r}{r + \lambda_m} \quad \textit{and} \quad C_1 = \frac{r - (\alpha B)^{\frac{\alpha}{1-\alpha}}}{r + \lambda_m - (\alpha B)^{\frac{\alpha}{1-\alpha}}}.
$$

Willingness to pay

Definition

The willingness to pay $v = v(W, H, \mathcal{P}_0, \Delta)$ to avoid a permanent change $\Delta \in [\mathcal{P}_0, 1 - \mathcal{P}_0]$ in death risk exposure $\mathcal{P}$ solves

$$V(W - v, H, \mathcal{P}_0) = V(W, H, \mathcal{P}_0 + \Delta).$$

- ✓ $\Delta > 0$: Indifference between paying the equivalent variation $v > 0$ at base risk and not paying with higher death risk
- ✓ $\Delta < 0$: Indifference between receiving compensation $-v > 0$ and foregoing lower death risk exposure.

Proposition

The willingness to pay to avoid an admissible change $\Delta \in \mathcal{A}_m$ is :

$$v(W,H,\lambda_m,\Delta) = \left[1 - \frac{\Theta(\lambda_m^\star)}{\Theta(\lambda_m)}\right] N(W,H)$$

an increasing and concave function of Δ that is bounded by :

$$\underset{\Delta\in\mathcal{A}_m}{inf}\, v(W,H,\lambda_m,\Delta) = \left[1 - \frac{\Theta(0)}{\Theta(\lambda_m)}\right] N(W,H)$$

$$\underset{\Delta\in\mathcal{A}_m}{sup}\, v(W,H,\lambda_m,\Delta) = N(W,H)$$

with $\lambda_m^\star = \lambda_m + \delta$.

Value of a statistical life

Proposition

The value of a statistical life $v_s = v_s(W, H, \mathcal{P}_0)$ is the negative of the MRS between the probability of death and wealth computed from the indirect utility evaluated at base risk $\mathcal{P}_0$:

$$v_s = - \left. \frac{V_{\mathcal{P}}(W, H, \mathcal{P})}{V_W(W, H, \mathcal{P})} \right|_{\mathcal{P}=\mathcal{P}_0} .$$

and is given by

$$v_s(W, H, \lambda_m) = \frac{1}{A(\lambda_m)} N(W, H)$$

where $A(\lambda_m)$ is the MPC and N the net total wealth.

Equivalently, the VSL is also the marginal willingness to pay :

$$v_s(W, H, \mathcal{P}_0) = \left. \frac{\partial v(W, H, \mathcal{P}_0, \Delta)}{\partial \Delta} \right|_{\Delta=0} = \lim_{\Delta \to 0} \frac{v(W, H, \mathcal{P}_0, \Delta)}{\Delta}.$$

Theoretical VSL vs Empirical VSL

Definition

The empirical value of a statistical life, $v_s^e = v_s^e\,(W, H, \mathcal{P}_0, \Delta)$ is given by :

$$v_s^e(W, H, \mathcal{P}_0, \Delta) = \frac{v\,(W, H, \mathcal{P}_0, \Delta)}{\Delta}$$

for small increment $\Delta = 1/n$ where n is the size of the population considered.

- As $\Delta \to 0$, $v_s^e(W, H, \mathcal{P}_0, \Delta) \simeq v_s(W, H, \mathcal{P}_0)\,;$
- The bias $v_s^e - v_s$ depends on the curvature of the WTP and Δ.

Gunpoint value of life

Proposition

The gunpoint value $v_g = v_g(W, H, \mathcal{P}_0)$ is the WTP to avoid certain, instantaneous death and is given by :

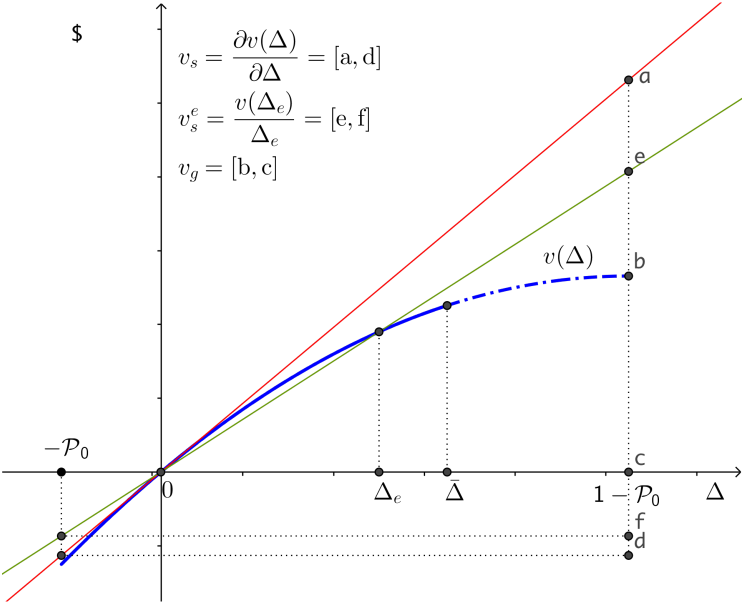
$$V(W - v_g, H, \mathcal{P}_0) = V^m$$

where V^m is the utility at death, and is given by

$$v_g(W, H) = N(W, H) \equiv W + \frac{y - a}{r} + BH.$$

- ✓ Unless y/r is large, $v_g(W, H) - v_h(W, H, \lambda_m) \geq 0$;
- ✓ $v_g(W, H) = A(\lambda_m) v_s(W, H, \lambda_m) < v_s(W, H, \lambda_m)$;
- ✓ $g(c_t - a) = g(v_{s,t}) = g(v_{g,t})$.

To summarize



4. Structural estimation

- Econometric model

$$\mathbf{Y}_j = \mathbf{B}(\theta)\mathbf{X_j} + \mathbf{u_j}$$

where

$$\mathbf{Y}_j = [c_j, \pi_j, x_j, I_j, Y_j]'$$
$$\mathbf{X}_j = [1, W_j, H_j]$$

- Data : PSID 2013
  - ✓ Health : "Poor" to "Excellent" using self-reported status (household head).
  - ✓ Financial wealth = risky (stocks in publicly held corporations, mutual funds, investment trusts, private annuities, IRA's or pension plans) plus riskless assets (checking accounts plus bonds plus remaining IRA's and pension).

Estimation of structural parameters

Parameter	Value	Parameter	Value
a. Law of motion health			
α	0.6843	δ	0.0125
	(0.3720)		(0.0060)
ϕ	0.0136^c		
b. Sickness and death intensities			
λ_s	0.0347	λ_m	0.0283
	(0.0108)		(0.0089)
d. Preferences			
γ	2.8953	ε	1.2416
	(1.4497)		(0.3724)
a	0.0140^c	ρ	0.0500^c

Value of Statistical Life *vs* HK Value

Wealth quintile level	Health level				
	Poor	Fair	Good	Very Good	Excellent
	a. Value of Statistical Life v_s				
1	2 167 573	4 379 551	6 591 529	8 803 507	11 015 485
2	2 168 877	4 380 874	6 593 136	8 805 188	11 017 133
3	2 188 829	4 400 253	6 614 190	8 827 429	11 040 023
4	2 360 907	4 582 287	6 800 733	9 021 052	11 238 999
5	4 710 118	7 889 684	9 595 444	12 136 981	15 012 108
All					
- mean	8 351 519				
- median	8 803 507				
	b. Human Capital Value of Life v_h				
	251 968	323 127	394 287	465 446	536 606
All					
- mean	437 756				
- median	465 446				

Gunpoint Value of Life *vs* HK Value

Wealth quintile level	Health level				
	Poor	Fair	Good	Very Good	Excellent
	a. Gunpoint Value of Life v_g				
1	116 121	234 620	353 120	471 619	590 119
2	116 191	234 691	353 206	471 709	590 207
3	117 259	235 729	354 334	472 901	591 433
4	126 478	245 481	364 327	483 274	602 093
5	252 329	422 664	514 045	650 199	804 225
All					
- mean			447 405		
- median			471 619		
	b. Human Capital Value of Life v_h				
	251 968	323 127	394 287	465 446	536 606
All					
- mean			437 756		
- median			465 446		

5. Discussion
Explaining the discrepancy

- Disjoint theoretical and empirical frameworks ?
  ✓ Answer : No !

- Collective WTP *vs* individual WTP ?
  ✓ Answer : The individual MWTP (theoretical VSL) is well approximated by the collective WTP (empirical VSL)

- Diminishing MWTP ?
  ✓ Answer : Yes ! Strongly diminishing MWTP does explain that VSL is much larger than HK value and GPV of life.

Back to basics ?

- VSL should *not* be interpreted as the value of a given human being (Schelling, 1968)
  - ✓ Rather a local measure of a rate of substitution between wealth and life
  - ✓ Empirical VSL measures adequately an individual MWTP when changes are small
  - ✓ VSL is appropriate when computing a collective value on small *indiscriminate* reductions on mortality for which society will ultimately end up paying the costs (e.g., public's safety).
- HK and GPV appear the better alternatives for wrongful death litigation or curative *vs* terminal care decisions.

Extensions

- ✓ Endogenous mortality and morbidity

$$\begin{aligned}\lambda_m(H_{t-}) &= \lim_{\tau\to 0}\frac{1}{\tau}P_t\left[t < T_m \leq t+\tau\right] = \lambda_{m0}+\lambda_{m1}H_{t-}^{-\xi_m}\\ \lambda_s(H_{t-}) &= \eta+\frac{\lambda_{s0}-\eta}{1+\lambda_{s1}H_{t-}^{-\xi_s}}\end{aligned}$$

- ✓ Ageing : Time-varying parameters $\lambda_{m,t}$, $\lambda_{s,t}$, ϕ_t, δ_t or β_t.
- ✓ SHARE data
- ✓ Immortal Life Value : WTA a compensation to renounce to perpetual life

Results remain applicable and are robust.

6. Conclusion

Questions	Findings HK	VSL	GPV
Theoretical links?			
- Common framework	Dynamic human capital model		
- Willingness to pay	Incr. **concave, bounded**		
- Life valuations	ENPV(Div.)	MWTP	Limiting WTP* ENPV(excess cons.)
Role of primitives?			
- Technological	✓	✓	✓
- Depreciation risk	✓	✓	✓
- Mortality risk	✓	✓	x
- Preferences	x	✓	x
Robust. of reduced-form findings?	Yes, VSL $\gg$ HK $\approx$ GPV		
- Struct. est. life values	420 K$	8.35 M$	447 K$
Reasons for differences?			
- Different model, data?	No		
- Model specific?	No		
- Assumptions?	Yes, **curvature** of WTP		

Overview of Gunpoint Value

- Hicksian Equivalent Variation (EV) : Maximal willingness to pay (WTP) to avoid unfavorable event (death).
- Highwaymen question :

  > *What is the amount you would be willing to pay in order to survive in a credible "your money or your life" highwayman threat or, equivalently, how much would you value your own life ?*

- **Gunpoint Value of Life** (GPV), i.e. the equivalent variation that leaves the agent indifferent between remaining alive and *certain* death.

Return